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An Integrated Brain Function

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Abstract

The aim of this paper is to provide sheaf theoretic formulations of the functions of a brain based on the theory of temporal topos, which is often abbreviated as *t*-topos as developed in Kato [1-4]. Our methods come from T-topos theory, which has been studied for the purpose of quantum gravity treating the concept of gravity in microcosm by capturing the associated sheaf with space-time. As a device connecting the discrete notion of a presheaf to smooth space-time as a global object as observed in macrocosm, we have found the theory of sheaves to be remarkably appropriate. An interpretation of brain imaging results will be analyzed as a transition from local data to global information in terms of sheaf theoretic methods. Profound phenomena of organization and emergence will be viewed in the light of the notion of a sheaf.

Introduction

As a Human activity, minds have created mathematics. On the other hand, we can ask whether mathematics may be used express how our mind and body work. Actually, Ehresmann and Vanbremeersch [5] proposed a mathematical model for complex systems, like an internal organization consisting of components with interrelations, based on category theory. The notions from categories and sheaves have been most powerful and effective methods not only for quantum physics as Isham [6], Mallios-Zafiris [7] and Kato [4], but also for describing the structure of living systems and their dynamic behavior [8,9]. In describing the workings of organic systems that are not Turing computable, such as metabolism-repair and replication systems, relations or mutual dependence of parts of organic systems may also be described by the category theory [10]. If we include time as a dimension, anticipatory behavior also becomes the object of our study [11].

For our mind Edelman [12] explains, in evolutionary terms, consciousness in terms of the morphology of the brain. Consciousness results from brain as a complex system where the diversity and the regularity of its responses emerge in the absence of the control.

There are researches started in the computer science led by Abramsky at Oxford University. Abramsky [13] studies a unified model of various theories from quantum physics to natural environments, Coecke and Pulman [14] also does in particular in connection to artificial intelligence, linguistics and cognitive Science. They show how the same mathematical structures arise in various areas of classical computation and use of string diagrams derived from quantum mechanics to gain insights into the nature of natural language [15].

Thus, there are numerous works on the applications of Category theory to human beings. All the activities mentioned above are relevant to our approaches via the methods of temporal topos theory.

The aim of this paper is to provide sheaf theoretic formulations of the functions of a brain based on the theory of temporal topos (which is abbreviated as *t*-topos in the following), developed in Kato [1-3]. The notion of a topos was originally developed by Alexandre Grothendieck in 1960's in Algebraic Geometry. By definition, a topos is the category of sheaves over a category

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with a Grothendieck topology, which is called a site. See Gelfand-Manin [16], Kashiwara-Schapira [17] or Kato [18] for the notion of a site. The concept of a sheaf was discovered by K. Oka in his work in Complex Analysis in several variables and by J. Leray in Algebraic Topology, both in 1940's. The first systematic formulation as sheaf cohomology was done during H. Cartan's seminar in 1950's. After the publication of the first paper in 1948 on category theory written by S. Eilenberg and S. Mac Lane, the notion of a category has been used in modern mathematics. However, it was Grothendieck who had developed those theories to the highest level. Applications to physics, especially for foundations of quantum theory and quantum gravity, were initiated by the topos theoretic school at Imperial College led by C. Isham. See Butterfield-Isham [19]. Temporal topos theory is a modified theory of a general topos theory. The most recent report on the development of the temporal topos theory can be found in Kato [4]. An interpretation of brain imaging results will be analyzed as a transition from local data to global information in terms of sheaf theoretic methods. Namely, profound phenomena of organization and emergence will be viewed in the light of the notion of a sheaf. For the microcosm level study of brain activities, we need a full account of temporal topos theory since quantum theory is involved as shown in Kato [1-4], where Heisenberg's uncertainty principle and wave-particle duality are relevant.

Our project on the formulations of the thinking processes began with the results in Kato-Nishimura [20] for two types of grasping concepts, and Kato-Nishimura [21] for thought processes[a]. This paper is the third in our series. In order to analyze brain functions, we will introduce two kinds of restrictions. For an active brain, we can consider the sub-activities of the whole brain restricted to smaller parts as frontal, parietal, temporal, and occipital lobes. Note that some brain activity requires only a few lobes rather than the entire brain. We also need to consider another notion of a restriction, which is less obvious and less known outside mathematical fields. For this second notion of a restriction, we need temporal topos methods as developed in Kato [3] as the foundations for temporal topos theory, Kato [1], and Kato [2] for its relativistic version. The most updated version by the first author can be found in Kato [4]. The second notion of the restriction (morphism) with respect to a generalized time period in the t-site is to express a local datum. In this paper, we will focus on the formulation of macrocosmic level subdivisions, e.g., lobe level subdivisions rather than a microcosm where t-topos theoretic methods, as done in Kato [3,4], for the quantum level are required. However, our formulation has potential to extend the methods and results in this paper to finer subdivisions of a brain into, e.g., neurons or an ultimate quantum level. Such a finer subdivision of a brain corresponds to the finer decomposition of the (pre) sheaf associated with the brain. See, e.g., Kato [3] for this restricted notion of a presheaf adapted for t-topos theory. For a transition from local to global, we need to formulate a certain kind of pasting mechanism in terms of categorical notions under those restrictions for a decomposition of a presheaf and a covering of an object in the t-site. We experience such an emergence as we hear just one or two notes of a well-known music piece. It is crucial to formulate such a notion as "recognition" and "understanding" in a precise way in terms of sheaf theory in the transition from local to global. See our forthcoming paper Kato-Nishimura on *Memory, Understanding, Discovery and DejaVu*, where a comprehensive treatment of cognitive activities in terms of t-dual topos theory.

[a]Consult Nishimura, et al. [22] for motivations of this classification. See Kato-Nishimura [20] for Categorical formulations of visual and verbal activities, and Citti, Petitot and Sarti [23] for the works on the mathematical analysis on the visual activities.

Restrictions and Subdivisions

We have introduced the sheaf theoretic methods in earlier papers Kato-Nishimura [20,21] for an analysis on brain activities. Recall that for a sheaf associated with a brain, the objects of a temporal site play a role in describing the changing states of such a sheaf. (See, e.g., Kato [3] for the notion of a temporal site or t-site).

In our approach in terms of t-topos, we associate a presheaf m for an entity M (for example, a particle like an electron, or human being). A measurement of M can be phrased as morphism from m to another presheaf p, which is the associated presheaf with an observer P. As a parameter indicating the states of the observer and the observed, there is a notion of t-site. As explained in Kato [1-4], we associate a presheaf m for every entity M. Such a presheaf m is said to be the associated presheaf of M. Namely, for a particle M in macrocosm or microcosm, there exists a presheaf m representing M. Since the usual time is represented by the time-associated t-site, an object of the t-site is referred to as a generalized time period. See Kato [1-3] for more detail discussion about t-site. When a presheaf m is said to be not reified, i.e., not defined, at any object of a t-site S, the presheaf m is said to be in an *ur-wave state*, or *wave ur-state,* and when m is reified at U of the t-site S, m is said to be in an *ur-particle state* or *particle ur-state* over U. However, the presheaf associated with an active brain is also a *sheaf* in the following sense. The condition for a presheaf to be a sheaf is the following. When local section data agree on the overlaps, the local sections can be pasted together to obtain a unique global section. To be more precise, the following sequence is exact

$$B(U)\xrightarrow{\rho_i}\prod_{i\in I}B\,(U_i)\;\underset{\rho^j_{j,i}}{\overset{\rho^i_{i,j}}{\rightrightarrows}}\;\prod_{i,j\in I}B\,(U_i\times U_j)\qquad(1.0)$$

for a covering $\{U\xleftarrow{i_i}U_i\}_{i\in I}$ when presheaf ***B*** actually is a sheaf. That is, exact sequence (1.0) can be read as: for $o_i\in B(U_i)$ and $o_j\in B(U_j)$, if $\rho^i_{i,j}(o_i)=\rho^j_{j,i}(o_j)$ holds (i.e., $o_i\in B(U_i)$ and $o_j\in B(U_j)$ agree on the categorical "intersection" $U_i\times U_j$, there exists a unique $o\in B(U)$ to match up with the given local data, i.e., $\rho_i(o)=o_i$ for each $i\in I$.

We will recall briefly several notions from our *t*-topos theoretic methods before we begin our main theme.

By definition, a presheaf is a contravariant functor from a category to a category. In our case, we choose site *S* as the domain category. Note that a site is a category with a Grothendieck topology. See Gelfand-Manin [16], Kashiwara-Schapira [17] or Kato [18] for the definition of a Grothendieck topology. As a codomain category of a functor as a presheaf, we often choose the category ((*Set*)) of sets, or the category ((*Ab*)) of abelian groups in which cohomologies can be defined. For our *t*-topos case, the codomain category is a product category where the measurement takes place. See also Kato [1,2] for a product category for *t*-topos theory. Let *S*^ be the category of presheaves on the site *S*. For any presheaf *B* on *S* and for any morphism $V\xrightarrow{f}U$ in *S* (i.e., for a presheaf *B* and $V,U\in Ob(S)$), we have the induced morphism

$$B(V)\xleftarrow{B(f)}B(U)\qquad(1.1)$$

Notice that the direction of the arrow $V\xrightarrow{f}U$ in *S* is reversed in (1.1), which is the result of the contravariant functor *B*. Note also that *B*(*f*) would be the induced restriction map from the inclusion map $i:V\to U$ if the site is a classical topological space. For our purpose, we need the notion of a site rather than a topological space since we need more morphisms than just one (the inclusion map) between objects of a site. For example, not every morphism in *S* is a linearly *t*-ordered morphism, which induces the usual time order "before and after". See Kato [3,4] for *t*-topos version of space-time for quantum physics.

A subdivision of a brain into e.g., lobes or even finer neurons, corresponds to a decomposition of a sheaf into subsheaves. Namely, for a sheaf ***B*** associated with a brain, we consider a direct product in the category *S*^ of presheaves:

$$B=\prod_{k\in J}B_k\xrightarrow{\pi_k}B_k\qquad(1.2)$$

Where π_k is the *k-th* projection from *B* onto B_k. If a subdivision of *B* into frontal, parietal, temporal, and occipital lobes is considered, then the index set ***J*** consists of four elements, i.e., *k* = 1, 2, 3, 4. Then subsheaves $\{B_k\}_{1\le k\le 4}$ are associated with those four lobes. In general, we say that a sheaf *B* is reified if there exists an object *U* of *S* so that *B*(*U*) is defined. Note also that even if global sheaf *B* is reified with *U*, local subsheaves B_k need not be reified over the same *U*. See Kato [3] for a general discussion of this matter. However, since we are dealing with a macrocosmic object, in what will follow, we assume $B_k(U)$ is defined whenever *B*(*U*) is defined. One could have further decomposition of subsheaf B_k into even elementary particles. Namely, we will not discuss any quantum mechanical issues in this paper. For the discussion on the foundations of quantum theory in terms of *t*-topos theory, see Kato [1-4] where quantum wave-particle duality, uncertainty principle and quantum entanglement are formulated in terms of sheaves and categories. This assumption of global *B* and a part B_k being defined on the same *U* is reasonable because: when the entire brain is measured over an object *U* of *t*-site *S* (e.g., by a brain imaging method), the states of the subdivided parts (corresponding to B_k) of the brain can be measured simultaneously. (At the quantum mechanical level, this simultaneity assumption is not accurate in terms of *t*-topos theory. However, as we have mentioned earlier, our focus is not microcosmic states where quantum physical considerations must be employed). Consequently, we have the induced morphism over *U* as follows.

$$B(U)\xrightarrow{\pi_k(U)}B_k(U)\qquad(1.3)$$

Which is the restriction of the state of the whole brain sheaf *B* determined by *U* to the state of a subdivision subsheaf B_k of *B*. Namely, for a common generalized time period *U*, we have the restriction of a sheaf *B* to a subsheaf B_k. Let *P* be the presheaf associated with an observer. When a local ur-particle state $B_k(U)$ of a whole *B*(*U*) is measured by *P*(*U*), we have a morphism

$$B_k(U)\xrightarrow{s_U}P(U)\qquad(1.4)$$

Then we can compose those morphisms in (1.3) and (1.4) to get

$$\begin{array}{ccc}B(U)&\xrightarrow{\pi_k(U)}&B_k(U)\\&\searrow^{s_U\circ\pi_k(U)}&\downarrow^{s_U}\\&&P(U)\end{array}\qquad(1.5)$$

which should be read as: the image of $s_U\circ\pi_k(U):B(U)\xrightarrow{s_U\circ\pi_k(U)}P(U)$ is the information of the state of *B*(*U*) obtained by *P*(*U*) via the sub-state $B_k(U)$.

Next we will consider the second type of a restriction for our formulations of brain activities in terms of sheaves and categories. This type of a restriction comes from a covering of an object *U* of the *t*-site *S*. As indicated earlier, such an object *U* of a *t*-site plays the role of a parameter (e.g., time) for *t*-topos theory. An object *U* of a *t*-site *S* is called a *generalized time period*, as a parameter for *t*-topos theory. Note that in our approach of *t*-topos, the objects of the *t*-site play a role in describing the changing states of (pre)sheaves. For an object U of the t-site S, let

$$\left\{U \xleftarrow{\lambda_i} U_i\right\}_{i\in I} \quad (1.6)$$

See Gelfand-Manin [16], or Kato [18] for the notion of a covering family of an object of a site which is a generalization of a covering for a topological space. For a morphism $U \xleftarrow{\lambda_i} U_i$ of the site S, we have the canonically induced morphism (the restriction)

$$B(U) \xrightarrow{B(\lambda_i)} B(U_i) \quad (1.7)$$

Which is the second kind of restriction mentioned earlier. Namely, the restriction (1.7) is the usual canonically induced morphism by $U \xleftarrow{\lambda_i} U_i$.

Remark: For a covering $\left\{U \xleftarrow{\lambda_i} U_i\right\}_{i\in I}$ as in (1.6), the morphism $\{\lambda_i\}_{i\in I}$ need not be linearly t-ordered. Recall that a morphism $U \xrightarrow{\lambda} U'$ is linearly t-ordered when in the usual time notion $t \sim \tau(U)$ precedes $t' \sim \tau(U')$. That is, the usual times t and t' corresponding to $\tau(U)$ and $\tau(U')$, respectively, have the linear order relationship $t \le t'$. See Kato [3] for the definition of a linearly t-ordered morphism.

Definition 1.1: Let B be the sheaf associated with a brain defined over a site S to the category ((*Set*)) of sets and let $B = \prod_{k\in J} B_k$ be a decomposition of into subsheaves $\{B_k\}_{k\in J}$ of B. Then the decomposition $B = \prod_{k\in J} B_k$ is said to be *coherent with respect to U* if the following sequence is exact

$$B(U) \longrightarrow \prod_{k\in J} B_k(U) \rightrightarrows \prod_{k,j\in J} B_k(U) \cap B_j(U) \quad (1.8)$$

Where all the horizontal morphisms are (1.3) - type restriction maps in the category ((*Set*)) of sets. The above exact sequence means the following: for $s_k \in B_k(V)$ and $s_j \in B_j(U)$, if $s_k = s_j$ over $B_k(U) \cap B_j(U)$, then there exists a unique global $s \in B(U)$ satisfying $\pi_k(U)(s) = s_k$ for each $s_k \in B_k(U)$ by the restriction morphism in 1.7.

Next we will consider an entire brain and its associated sheaf B. Let partial information $s_i \in B(U_i)$ for each $i \in I$, as a section s_i of sheaf B over U_i, be given for the (ur-particle) state $B(U_i)$ of the brain.

Definition 1.2: A presheaf B is said to be *coherent* when for a covering $\left\{U \longleftarrow U_i\right\}_{i\in I}$ and for $s_i \in B(U_i)$ and $s_l \in B(U_l)$, if $s_i = s_l$ over $U_i \times_U U_l$, there exists a unique $s \in B(V)$ satisfying $s = s_i$ over U_i for all $i \in I$.

Namely, a presheaf B is coherent with respect to the covering if B is a sheaf, i.e., B satisfies the exact sequence (1.0).

On the other hand, the restriction in (1.3) is related to the physical restriction of the entire brain activity of $B(U)$ to the sub-state (ur-sub-state) of $B(U)$.

Local-Global Mechanism of a Sheaf Associated with a Brain

Theorem 2.1: Let B be a sheaf associated with a brain. Let $B = \prod_{k\in J} B_k$ and $\left\{U \xleftarrow{\lambda_i} U_i\right\}_{i\in I}$ be a decomposition of B into subsheaves $\{B_k\}$ and a covering of U, respectively. Assume that for U and for the covering of U, the decomposition $B = \prod_{k\in J} B_k$ is coherent in the sense of *Definition 1.1*, where $B(U)$ is reified. Assume also that for the decomposition of B and the covering of $U, \{B_k(U_i)\}_{k\in J, i\in I}$ are reified. Then for a given local datum

$$\left\{s_i^k \in B_k(U_i)\right\}_{k\in J, i\in I} \quad (2.1)$$

there exists a unique global $s \in B(U)$ satisfying the double restrictions $s_i^k = \left(\pi_i^k(U_i) \circ B(\lambda_i)\right)(s)$, where

$$B(U) \xrightarrow{B(\lambda_i)} B(U_i) \xrightarrow{\pi_i^k(U_i)} B_k(U_i). \quad (2.2)$$

Proof In order to construct such a unique $s \in B(U)$, we only need to observe the following commutative diagrams:

$$\begin{array}{ccc} & B(U_i) & \\ \swarrow & & \searrow \\ B_k(U_i) & & B_h(U_i) \\ \searrow & & \swarrow \\ & B_k(U_i) \cap B_h(U_i) & \end{array} \quad (2.3)$$

and

$$\begin{array}{ccc} & B(U) & \\ \swarrow & & \searrow \\ B(U_i) & & B(U_j) \\ \searrow & & \swarrow \\ & B(U_i \times_U U_j) & \end{array} \quad (2.4)$$

The above diagrams should be read as follows: For $s_i^k \in B_k(U_i)$ in (2.3) satisfying the condition of the exactness of the sequence (1.8), there exists a uniquely determined $s_i \in B(U_i)$. This is because the decomposition $B = \prod B_k$ is coherent with respect to U_i. For $s_i \in B(U_i)$ in (2.4)^k since the coherence of B coincides with the notion of a sheaf, we get a uniquely determined global $s \in B(U)$.

Conclusion

We have formulated explicitly in this paper the connection between local data and the global information in terms of t-topos. The local image of a local activity of a brain does not provide the global information, i.e., what a thought is about. By gluing the local data in the sense of sheaf theory, we obtain the global information where the two notions of restrictions described in this paper play significant a role.

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References

1. Kato GC (2004) Elemental principles of temporal topos. Europhysics Letters (EPS) 68: 4.
2. Kato GC (2005) Elemental t. g. principles of relativistic t-Topos. Europhysics Letters (EPS) 71: 2.
3. Kato GC (2013) Elements of temporal topos. Abramis Academic.
4. Kato GC (2017) Topos theoretic approach to space and time in treatise space time and the limits of human understanding. In: Wuppuluri, Shyam, Ghirarlo, Springer, New York.
5. Ehresmann A, Vanbremeersch JP (1987) Hierarchical evolutive systems: A mathematical model for complex systems. Bulletin of Mathematical Biology 49: 13-50.
6. Chris J Isham (2011) Topos methods in foundations of physics. In: H Halvorson, Deep Beauty: Understanding the Quantum World through Mathematical Innovation. New York: Cambridge University Press, 1-24.
7. Mallios A, Zafiris E (2016) Differential sheaves and connections. World Scientific.
8. Lakoff G (1987) Women, fire, and dangerous things: what categories reveal about the mind. University of Chicago Press, Chicago.
9. Lakoff G, Núñez RE (2000) Where mathematics comes from: how the embodied mind brings mathematics into being. Basic Books, 24: 75-78.
10. Louie AH (2013) The reflection of life: functional entailment and imminence in relational biology. International Journal of General Systems, Springer, New York.
11. Rosen R (1985) Anticipatory systems. Pergamon Press, Oxford, 405-409.
12. Edelman G (2006) Second nature: brain science and human knowledge. Yale University Press, New Haven and London.
13. Abramsky S (2015) Contextual semantics: from quantum mechanics to logic, databases, constraints, and complexity.
14. Coecke B, Pulman SG (2011) A unified model of compositional and distributional semantics: Theory and applications. EPSRC.
15. Abramsky S, Coecke B (2007) Physics from computer science. International Journal of Unconventional Computing 3: 179-197.
16. Gelfand SI, Manin YI (1996) Methods of homological algebra. Springer.
17. Kashiwara M, Schapira P (2006) Categories and sheaves. Springer.
18. Kato GC (2006) The Heart of cohomology. Springer, 1-11.
19. Butterfield J, Isham CJ (2001) Spacetime and the philosophical challege of quantum gravity.
20. Kato GC, Nishimura K (2013) Grasping a concept as an image or as a word. Journal of Scientific Research and Reports 2: 682-691.
21. Kato GC, Nishimura K (2015) What is a thought process? Journal of Integrated Creative Studies 1: 1-7.
22. Nishimura K, Aoki T, Inagawa M, et al. (2015) Brain activities of visual thinkers and verbal thinkers: A MEG Study. Neurosci Lett 594: 155-160.
23. Citti G, Petitot J, Sarti A (2009) Editorial for special issue on neuromathematics of vision. J Physiol Paris 103: 1-3.